

Remarks on E11 approach

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Abstract

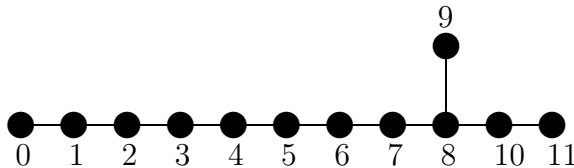
We consider a few topics in E_{11} approach to superstring/M-theory: even subgroups (Z_2 orbifolds) of E_n , $n=11,10,9$ and their connection to Kac-Moody algebras; EE_{11} subgroup of E_{11} and coincidence of one of its weights with the l_1 weight of E_{11} , known to contain brane charges; possible form of supersymmetry relation in E_{11} ; decomposition of l_1 w.r.t. the $SO(10,10)$ and its square root at first few levels; particle orbit of $l_1 \ltimes E_{11}$. Possible relevance of coadjoint orbits method is noticed, based on a self-duality form of equations of motion in E_{11} .

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1 Introduction

One of the recent ideas on a hidden structures in M-theory is that of a hidden E_{11} and/or E_{10} Kac-Moody Lie algebra symmetry [1, 2, 3], generalizing an U-duality of compactified superstrings/M-theory. Appearance of E series of Lie algebras as a symmetry of supergravity theories started from the discovery of E_7 as a symmetry (of equations of motion) of maximal four-dimensional supergravity [4]. Afterwards, E_n type algebras, with Dynkin diagrams given below were discovered to appear in compactifications of 11d supergravity and superstrings to $11 - n$ dimensions, including E_9 affine algebra in 2d [5] and E_{10} in a 1-dimensional reduction in a form of a particle motion in an E_{10} Weyl chamber (see [6]).



Dynkin diagrams of E_{12-n} are given by nodes $n, n+1, \dots, 11$ (1)

It was the main idea of [1] to consider E_{11} as a symmetry of M-theory. Formally, its Dynkin diagram appears in U-duality considerations of M-theory compactified to 0 dimensions [7]. The point is that E_{11} was suggested in [1] as a symmetry of opposite extreme - completely uncompactified theory. There is a number of arguments in favor of this idea: the field content of model of [1] recovers (the first levels of) the M-theory, the T-duality between IIA and IIB theories appears to be a simple property of E_{11} Dynkin diagram, the brane charges seem to fill in one of the fundamental representations of E_{11} , and others. The accompanied difficulties can be seen from this last observation: since in the usual approach the space-time is associated with point-like charge, which now is the part of E_{11} irrep, the E_{11} covariance requires substitution of space-time with duals of all (infinite number of) brane charges, which apparently is not a standard situation in field or string theories.

The similar E_{10} suggestion [3] is much more compact and more precise. It deals with 1d sigma model, instead of infinite-dimensional one, namely that

based on a coset of E_{10} group, with fields depending on one parameter in a reparametrization invariant way. The price is the loss of (at least explicit) Lorentz invariance, since the space-time is introduced in this approach by assumption that coefficients of expansion of fields over coordinates appear on the higher levels of algebra (the number of which is infinite, due to the Kac-Moody nature of E_{10}).

The E_{11} model will be our main object of study in this paper, we will present a few results on different aspects of the topic.

According to [1, 2], the hypothesis is that sigma model over coset space E_{11}/K_{11} gives some description of M-theory. To define K_{11} we first introduce notations for Kac-Moody Lie algebra with Cartan matrix A_{ij} . It's the Lie algebra with generators h_i, e_i, f_i and relations:

$$[h_i, e_j] = A_{ij}e_j \quad (2)$$

$$[h_i, f_j] = -A_{ij}f_j \quad (3)$$

$$[e_i, f_j] = \delta_{ij}h_j \quad (4)$$

$$ad(e_i)^{(1-A_{ij})}e_j = 0, \quad (5)$$

$$ad(f_i)^{(1-A_{ij})}f_j = 0 \quad (6)$$

All other generators should be obtained from these prime generators by all possible multiple commutators, factorized over relations (2)-(6). Matrix A_{ij} has the following properties: $A_{ii} = 2$, A_{ij} are non-positive integers such that from $A_{ij} = 0$ follows $A_{ji} = 0$. A_{ij} alternatively and equivalently can be presented by Dynkin diagram, with simple rules of equivalence. For example, for symmetric A_{ij} with non-diagonal entries 0 or -1 Dynkin diagram is given by nodes equal to dimensionality of A_{ij} , nodes i and j are connected by simple line if and only if $A_{ij} = -1$.

Next, Chevalley involution of an arbitrary Kac-Moody algebra is the Lie algebra automorphism

$$h_i \rightarrow -h_i \quad (7)$$

$$e_i \rightarrow -f_i \quad (8)$$

$$f_i \rightarrow -e_i \quad (9)$$

Subalgebra K_{11} consists of generators invariant under Chevalley involution, it is generated by elements $e_i - f_i$. K_n is a key object for the discussion of supersymmetry, also, see below. The study of K_9 see in [8].

In the body of paper, in Section 2 we consider the even (Z_2 invariant, see (11)-(13)) subalgebras of E_n algebras. From the M-theory viewpoint they are relevant, particularly, for orbifold considerations [9]. Moreover, for finite dimensional algebras E_7, E_8 the even subalgebras actually coincide with Chevalley-invariant subalgebras K_7, K_8 . This statement actually extends to all finite-dimensional algebras where corresponding K_n has rank n . In [10] it was suggested that this coincidence extends to Kac-Moody algebras $E_n, n = 9, 10, 11$, however, it seems that this assumption is not correct. Which concerns a description of even subalgebras, our claim is that the even roots of E_{11}, E_{10} and E_9 coincide with all roots of EE_{11}, DE_{10} and $D_8^{(1)}$, respectively. For E_{11} that is supported by numerical calculations, for E_{10} and E_9 that is proved below, for E_9 that gives the complete coincidence of algebras, since the multiplicities of imaginary roots coincide, also.

Next Section 3 considers the weights of fundamental representations of EE_{11} , introduced in previous Section 2. Observation is that one of its fundamental weights coincides with weight of l_1 - first fundamental weight of E_{11} , known to contain the brane charges [11]. This kind of considerations are aimed to discussion of possible supersymmetry relation in E_{11} theory:

$$\{Q, Q\} = Z \quad (10)$$

In usual supersymmetric theories the supercharges Q should be a representations of both compact subgroup K_n and Lorentz group. E.g., 3d compactification of 11d supergravity gives a $E_8/SO(16)$ supersymmetric sigma-model [12], with supercharges in spinor representation of Lorentz group and vector of $SO(16)$. In E_{11} approach these two groups are joined into K_{11} group. From the other side, the anticommutator of supercharges gives the brane charges Z , which, as argued in [11], is l_1 , the irreducible representation of E_{11} . So, roughly speaking, the symmetric square of Q representation of K_n gives a fundamental irrep of E_{11} . More precisely, one can imagine that some Klebsh-Gordon coefficients can enter in r.h.s. of (10), so l_1 is one of irreps, appearing in the r.h.s after decomposition to irreducible representations. The symmetric square of highest-weight representation gives, particularly, the irrep with doubled highest weight. So, we see that the coefficient 2 is missing in abovementioned statement of coincidence of one of weights of EE_{11} with weight of l_1 , saying least. Nevertheless, the coincidence of highest weights can signal on some relevance of EE_{11} . The attempt to introduce a supersymmetry in the E_{11} approach was done in [13], where the Killing spinor

equations are constructed, and fermionic generators are introduced into a part of G_{11} algebra, which was an intermediate step in construction of E_{11} in [1]. It seems, however, that these results are not relevant for supersymmetrization of E_{11} itself, (10), due to the few reasons, one of which is that group, considered in [13], includes momenta which is not the part of E_{11} . Further discussion of problems of [13] see in Section 4

The discussion of supposed susy relation (10) is continued in the next Section 4, where we study the expansion of first fundamental weight l_1 of E_{11} w.r.t. the levels of root e_{11} of E_{11} , (rightmost root of diagram (1)) and show that the subset of representations at first three levels can be obtained as a symmetric square of representations of corresponding compact subgroup $SO(10) \times SO(10)$. This result is another face of similar phenomena found in [14].

Section 5 is devoted to the study of hypothesis that finally symmetry group should be extended to semidirect product of l_1 and E_{11} [15]. We calculate the little group for particle orbit, i.e. for a given point in the space l_1 of brane charges, when all charges are zero, except the particle one, we calculate its stabilizer in E_{11} . It appears to have an explicit description in terms of basic generators of E_{11} .

Conclusion contains the resume of results and ways of their development, particularly, possible relevance of coadjoint orbits of E_{11} is discussed.

2 Even subalgebras of E_n

We consider involution of E_n algebras, given by

$$h_i \rightarrow h_i \tag{11}$$

$$e_i \rightarrow -e_i \tag{12}$$

$$f_i \rightarrow -f_i \tag{13}$$

The corresponding invariant subalgebra is given by generators h_i and those of even power of e_i or f_i . Let's denote that by $Z_2(E_n)$. Study of this subalgebras is relevant for Z_2 orbifolds [9]. For finite dimensional Lie algebras g with the property that rank of CSA of K_n (Chevalley-invariant subalgebra) is maximal, i.e. equal to n , K_n coincide with $Z_2(g)$,

$$K_n(g) = Z_2(g) \tag{14}$$

It is suggested in [10] that (14) extends to Kac-Moody algebras. However, the problem is in Cartan subalgebra. Although one can find [10] a lot of commuting generators, even with hermiticity properties, their diagonalizability is questionable, since one can show that they have to be diagonalized in the infinite subspaces ¹.

As proposed in [14] and [16], the study of bilinear invariant forms can shed light on a problem of connection of compact subalgebra with Kac-Moody algebras. Particularly, in [16] is shown, that special contravariant Hermitian bilinear form is positively defined on K_n , and this leads to a conclusion that K_n is not a semisimple Kac-Moody algebra.

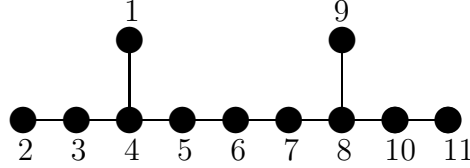
Now we shall try to construct even roots of E_{11} from some basic even roots. We would like to introduce the subalgebra of E_{11} generated by CSA and following generators (Lie algebra commutators are implied) and their opposite roots partners:

$$\begin{aligned} a_1 &= e_7 e_8 e_9 e_{10}, a_2 = e_1 e_2, a_3 = e_3 e_4, a_4 = e_5 e_6, \\ a_5 &= e_7 e_8, a_6 = e_{10} e_{11}, a_7 = e_8 e_9, a_8 = e_6 e_7, \\ a_9 &= e_8 e_{10}, a_{10} = e_4 e_5, a_{11} = e_2 e_3, \end{aligned} \tag{15}$$

Definition of a_1 is actually unique, up to overall sign, since although Lie brackets can be arranged in different ways, results coincide. Roots of (15) are real. One can find the corresponding Cartan matrix and Dynkin diagram:

$$EE_{11} = \begin{array}{cccccccccccc} 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{array} \tag{16}$$

¹We are indebted to H.Nicolai for e-mail correspondence stressing the importance of diagonalizability of Cartan generators

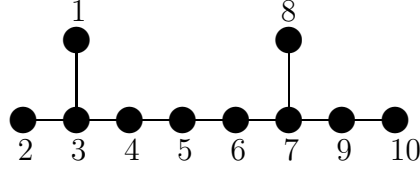


Dynkin diagram of EE_{11} algebra (17)

where simple roots in (17) are enumerated in agreement with (16). One can construct the corresponding abstract Kac-Moody algebra, we denote it by EE_{11} since it contains two E type tails, and this notation is similar to that of hyperbolic algebras - AE, BE, CE, DE. Since roots (15) are real, according to [18], algebra (17) is isomorphic to above defined subalgebra in E_{11} . Our hypothesis is that this algebra has the same roots as $Z_2(E_{11})$, i.e. all even roots of E_{11} . Statement seems to be simple, nevertheless, we were not able to prove it algebraically, due to unknown structure of roots system. Instead we checked that up to level 146 by the help of computer program (available upon request), which generates the roots for an arbitrary input Dynkin diagram. The number of roots up to the level 146 (inclusively) is 19661788 (without counting multiplicity), so coincidence is considerable. Since multiplicity of real roots is one, this statement means that these two algebras coincide at least in the sector of real roots. However, as shown in [9] for similar considerations for E_{10} (see below), for imaginary roots there is difference in multiplicities in the case of $Z_2(E_{10})$ and DE_{10} , so we are confident in the same statement for E_{11} .

Similar statements (on the coincidence of even roots and roots of subalgebras of simple even roots) for E_{10} and E_9 can be proved. Corresponding composite roots (generators) and Dynkin diagram for $Z_2(E_{10})$ are:

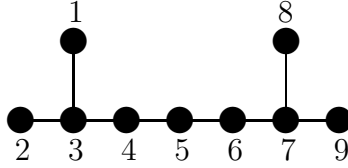
$$\begin{aligned}
 a_1 &= e_6 e_7 e_8 e_9, a_2 = e_2 e_3, a_3 = e_4 e_5, a_4 = e_6 e_7, a_5 = e_9 e_{10}, \\
 a_6 &= e_7 e_8, a_7 = e_5 e_6, a_8 = e_7 e_9, a_9 = e_3 e_4, a_{10} = e_1 e_2
 \end{aligned}
 \tag{18}$$



Dynkin diagram of DE_{10} algebra (19)

For E_9 :

$$\begin{aligned} a_1 &= e_1 e_2, a_2 = e_5 e_6 e_7 e_8, a_3 = e_3 e_4, a_4 = e_5 e_6, \\ a_5 &= e_8 e_9, a_6 = e_6 e_7, a_7 = e_4 e_5, a_8 = e_6 e_8, a_9 = e_2 e_3 \end{aligned} \quad (20)$$



Dynkin diagram of $D_8^{(1)}$ algebra (21)

Both for E_{10} and E_9 coincidence of roots follows from two facts. First, root lattices coincide [9], namely, even root sublattice of E_{10} (E_9) coincide with lattice of DE_{10} (19) ($D_8^{(1)}$ (21)). Second, description of all roots is given in ([19], p.67), in terms of lattices - all real roots are all those elements of lattice with square equal two, and all other roots (i.e. imaginary ones) are all those elements of lattice with square less or equal to zero. For E_9 that means complete coincidence of algebras:

$$Z_2(E_9) = D_8^{(1)} \quad (22)$$

since multiplicities of imaginary roots coincide (multiplicities of $Z_2(E_9)$ are that of E_9 , which is 8, since that is an affine algebra $E_8^{(1)}$, and multiplicity of roots of $D_8^{(1)}$ is 8, also).

For E_{10} the problem of multiplicities is more complicated, and in [9] it is shown that actually multiplicities of $Z_2(E_{10})$ and DE_{10} are different. It would be interesting to describe that difference explicitly.

For E_{11} the coincidence of its even lattice of and that of EE_{11} can be proved, also, but it is not enough for coincidence of all roots, since for these non-hyperbolic algebras there is no similar description of roots.

Coincidence of even roots of E_{10} and all roots of DE_{10} also follows from statements on a level decompositions of these two algebras, proved in Section 4.2 of [17].

3 On a representations of EE_{11}

The weights of the fundamental representations of E_{11} . can be obtained by the rows of an inverse Cartan matrix $(EE_{11})^{(-1)}$ of (17). We would like to compare these weights with those of E_{11} :

$$E_{11}^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 3 & 4 & 2 \\ 0 & 0 & 2 & 4 & 6 & 8 & 10 & 12 & 6 & 8 & 4 \\ 1 & 2 & 3 & 6 & 9 & 12 & 15 & 18 & 9 & 12 & 6 \\ 2 & 4 & 6 & 8 & 12 & 16 & 20 & 24 & 12 & 16 & 8 \\ 3 & 6 & 9 & 12 & 15 & 20 & 25 & 30 & 15 & 20 & 10 \\ 4 & 8 & 12 & 16 & 20 & 24 & 30 & 36 & 18 & 24 & 12 \\ 5 & 10 & 15 & 20 & 25 & 30 & 35 & 42 & 21 & 28 & 14 \\ 6 & 12 & 18 & 24 & 30 & 36 & 42 & 48 & 24 & 32 & 16 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 11 & 16 & 8 \\ 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 16 & 20 & 10 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 8 & 10 & 4 \end{bmatrix} \quad (23)$$

The only subtlety is that rows of inverse Cartan matrix express weights in the basis of simple roots of a given algebra, so for comparison we should express both in the same basis, e.g. in a basis of simple roots of E_{11} . The expression of simple roots of EE_{11} through the simple roots of E_{11} is given in (15). So we should multiply the matrix $(EE_{11})^{(-1)}$ from the right by the transformation matrix

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (24)$$

and obtain

$$(EE_{11})^{-1}T = \frac{1}{8} \begin{bmatrix} 2 & 8 & 10 & 16 & 18 & 24 & 26 & 32 & 14 & 20 & 12 \\ -4 & 0 & 4 & 8 & 12 & 16 & 20 & 24 & 12 & 16 & 8 \\ 0 & 8 & 8 & 16 & 24 & 32 & 40 & 48 & 24 & 32 & 16 \\ 4 & 16 & 20 & 32 & 36 & 48 & 60 & 72 & 36 & 48 & 24 \\ 6 & 16 & 22 & 32 & 38 & 48 & 54 & 64 & 34 & 44 & 20 \\ 8 & 16 & 24 & 32 & 40 & 48 & 56 & 64 & 32 & 40 & 16 \\ 10 & 16 & 26 & 32 & 42 & 48 & 58 & 64 & 30 & 44 & 20 \\ 12 & 16 & 28 & 32 & 44 & 48 & 60 & 72 & 36 & 48 & 24 \\ 6 & 8 & 14 & 16 & 22 & 24 & 30 & 32 & 18 & 20 & 12 \\ 8 & 8 & 16 & 16 & 24 & 32 & 40 & 48 & 24 & 32 & 16 \\ 4 & 0 & 4 & 8 & 12 & 16 & 20 & 24 & 12 & 16 & 8 \end{bmatrix} \quad (25)$$

The second row of (25) and first row of (23) coincide. As mentioned in the introduction, this statement has some resemblance with the one that is implied by (supposed) supersymmetry relation (10), although it differs in few important points - coefficient 2 is missing, and, more important, EE_{11} is not a K_{11} , although can have similar properties. It is also worth recalling the discussion of [20], where the traces of needed phenomena were noticed, namely at certain dimensions certain brane charges are not only in the representation of corresponding susy algebra's automorphism group, but also combine into representations of U-duality group. For example, consider maximal susy algebra in 4 dimensions, obtained from reduction to 4d of 11d susy algebra.

The corresponding 4d algebra has $SU(8)$ as an automorphism group. Scalar central charges appear from a few sources: 7 from vector, 21 from membrane charge, and 28 from five-brane charge, altogether they combine into 56 of 4d U-duality group E_7 , which contains $SU(8)$ as its maximal compact subgroup. This field worths further study, the key element should be the theory of K_{11} group's representations.

4 l_1 expansion over $SO(10, 10)$

Lets's consider the expansion of both sides of supposed susy relation (10) over $SO(20)$ ($SO(10, 10)$ if reality properties taken into account) subgroup of E_{11} , obtained by removing most right root (number 11 on diagram (1)). The corresponding compact group is $SO(10) \times SO(10)$. This expansion can shed some light on whether such a relation can exist, at least on a first few terms of expansion. The level expansion of l_1 with respect to the generator e_9 was suggested in [14], where it was shown, that usual brane charges ($P_\mu, Z_{\mu\nu}, Z_{\mu_1\mu_2\mu_3\mu_4\mu_5}$) appear on first three levels of that expansion. As is well-known, one can fulfill relation (10) with Q as a spinors of corresponding compact group $SO(11)$. We would like to consider the same problem with expansion over e_{11} , with compact group $SO(10) \times SO(10)$, which was not supported by existence of any supersymmetric theories, but, from the other side, should exist, provided E_{11} hypothesis and relation (10) are correct.

Expansion over e_{11} goes with non-negative powers. This is clear when recalling an approach of [14] - in that paper l_1 is identified with the subspace of E_{12} , linear over e_0 . This subspace evidently is a representation of E_{11} , and actually the first fundamental representation, with e_0 as a highest weight vector. The same is true for the zeroth order over e_{11} - it is a representation with highest vector e_0 , since all $f_i, i > 0$ commute with e_0 and action of $h_i, i > 0$ on e_0 gives the only non-zero Dynkin label $p_1 = 1$, i.e. that of vector representation of $SO(20)$, 20. Next is linear over e_{11} representation of $SO(20)$. It is easy to understand, that the highest vector for that representation is unique, namely that given by unique nonzero commutator of e_0, e_1, \dots, e_{11} (except e_9). The only nonzero Dynkin label is $p_9 = 1$, i.e. that is one of two Weyl spinor representations of $SO(20)$, of dimensionality 512. Next is the second order over e_{11} representation, which is the last one we are interested in. It can be shown to be fifth-rank antisymmetric tensor Z_5 , with highest vector

$\sum n_i e_i$, $n_0 = \dots = n_5 = 1, n_6 = 2, n_7 = 3, n_8 = 4, n_9 = 2, n_{10} = 3, n_{11} = 2$. The corresponding Dynkin label is $p_5 = 1$ (other $p_i = 0$). We are interested in decomposition of these few first level representations over compact subgroup of $SO(10,10)$. The $\underline{20}$ is of course $\underline{10} + \underline{10}$, $\underline{512}$ is $(\underline{16}, \underline{16}) + (\underline{16}, \overline{\underline{16}})$ (chirality of $SO(20)$ representation choose chirality of $SO(10)$). Z_5 is decomposed in more or less clear way, according to all possible choices of belonging of indexes to two product $SO(10)$ groups: $Z_5 = Z_{5,0} + Z_{4,1} + Z_{3,2} + Z_{2,3} + Z_{1,4} + Z_{0,5} = (\underline{252}, \underline{1}) + (\underline{210}, \underline{10}) + (\underline{120}, \underline{45}) + (\underline{45}, \underline{120}) + (\underline{10}, \underline{210}) + (\underline{1}, \underline{252})$. Moreover, fifth index tensors should be decomposed into their chiral parts, i.e. into self-dual and anti-self-dual tensors of dimensionality 136. Here we imagine complex field of coefficients, neglecting reality properties of groups involved, their will tune themselves according to the choice of Chevalley subgroup.

After this preparatory work we can look for an answer on a possibility of taking a square root of l_1 , i.e. finding a representation of K_{11} , symmetric square of which contains l_1 . It appears that one can take the following combination of representation of $SO(10) \times SO(10)$. This combination should be considered as a decomposition of irrep of K we are seeking for, w.r.t. the $SO(10) \times SO(10)$:

$$(\underline{16}, \underline{1}) + (\underline{1}, \underline{16}) \quad (26)$$

Symmetric square of this representation is

$$(\underline{10}, \underline{1}) + (\underline{1}, \underline{10}) + (\underline{16}, \underline{16}) + (\underline{136}, \underline{1}) + (\underline{1}, \underline{136}) \quad (27)$$

This includes representations of first level, second level, and part of representations of third level, obtained above in decomposition of l_1 w.r.t. the powers of e_{11} . I.e. we find the square root of (part of the) first few levels of l_1 . Finally, the susy relation, in this approximation, can be represented in a well-known form

$$\{Q_\alpha, Q_\beta\} = Z_{\alpha\beta} \quad (28)$$

This is a standard form of 11d susy relation, where supersymmetry charge is 32 dimensional, giving in the r.h.s all possible 528 central charges. That means that we are dealing with the same $SL(32)$ invariant susy algebra, decomposed with respect to different subalgebras. The natural question is whether there exist $SO(10) \times SO(10)$ covariant supergravity theories, with

susy algebra (28). The $SL(32)$ invariance of (28) doesn't persist on the higher levels, since E_{11} does not have such a subgroup (see discussion in [21], where it was shown that antisymmetric tensor representations, precisely corresponding to those of $SL(32)$ decomposed w.r.t. the $SL(11)$ can be identified on a first 4 levels of K_{11}).

Let's mention a difference of previous considerations with approach of [13]. First, as mentioned in the Introduction, the groups considered are different - G_{11} of [13] includes momenta P_μ , which is not the part of E_{11} . Moreover, it is argued in [13] that two spinor generators are needed in supersymmetrization of G_{11} , on the basis that 11d conformal group $SO(2, 11)$ should be, finally, part of the symmetries considered. That statement stress that $SO(2, 11)$ group is not part of E_{11} , from the same fact of absence of momenta P_μ in E_{11} (as well as its conformal counterpart K_μ).

This discussion stress the problem of finding the generalization of E_{11} , which will include momenta and possibly the whole conformal group $SO(2, 11)$. As mentioned in Introduction, the generalization including momenta, actually the whole multiplet l_1 , is suggested in [14] as semidirect product of l_1 on E_{11} . Inclusion of conformal $SO(2, 11)$ will require further extension of this group.

Next remark concerns the possibility of continuation of above analysis to higher levels of e_{11} . The problem is in that there is no corresponding grading of desired representation of K_{11} . So it is not clear how to continue finding next terms of decomposition of that representation w.r.t. the $SO(10) \times SO(10)$.

It is worth mentioning here the connection between $SO(10) \times SO(10)$ subgroups of $SO(20) \subset E_{11}$, with EE_{11} . From the diagram of EE_{11} one can easily read off two $SO(10)$ subgroups, constructed from two D_5 sub-diagrams, that appear after removing the middle root (number six). One can easily understand, that they are the same $SO(10)$ subgroups of $SO(20)$ and E_{11} . It follows from the fact that eleventh root e_{11} of E_{11} enters in the sixth root of EE_{11} , only, and from the fact mentioned in Section 1, that compact subgroup of D_{10} can be represented as its even subgroup.

5 Particle orbit in $l_1 \ltimes E_{11}$

In the search of a space-time in the E_{11} approach, West [14] suggested to extend the symmetry group to semidirect product $l_1 \ltimes E_{11}$, which is similar to Poincare group. Then one has to consider a unitary representation of this

group, which can be constructed by Wigner's little group method, which is recently applied to construction of irreps of semidirect product of Lorentz and tensorial translations group [22]. The method requires choosing of the orbit of action of E_{11} on l_1 , and then construction of unitary irreps of little group - stabilizer of a given point of the orbit in the E_{11} . We will apply this method to particle orbit.

According to suggestion [14] representation l_1 contains all brane charges. Particularly, the decomposition of l_1 w.r.t. the $SL(11)$ subgroup of E_{11} starts from vector representation P_μ . Particle can be naturally defined as a configuration of brane charges when all of them are zero, except P_μ . Note that we are dealing with general linear GL_{11} group, which gives usual Lorentz $SO(11)$ after taking a Chevalley-invariant (=compact) subgroup of GL_{11} . Correspondingly, an arbitrary vector P_μ can be transformed into any other vector, so we can choose

$$P_\mu = (1, 0, 0, \dots) \quad (29)$$

Next, our aim is to define an orbit of this point under action of the whole group E_{11} . Desired orbit is a factor of E_{11} over L , the stabilizer of P_μ , i.e. subgroup of E_{11} , which leaves P_μ unchanged. So, our task is to find the subgroup L . Note that P_μ (29) is represented by just e_0 . So, we are seeking a stabilizer of e_0 in E_{11} . Evidently, among generators of E_{11} , commuting with e_0 , are E_{10} generators, constructed from elements e_i, h_i, f_i with indexes starting from 2. Next, among them are all generators of E_{11} with non-zero power of f_1 . It remains to consider generators of e_{11} with non-zero power of e_1 . They all have nonzero commutators with e_0 , which is evident from the rules of construction of roots - the scalar product of such generators with e_0 are nonzero negative integers (equal to the power of e_1), so real root α_0 can be added to the given root of E_{11} . So, the stabilizer of e_0 within E_{11} is $(E_{10}, (f_1)^n \dots (n > 0))$, where $(f_1)^n \dots (n > 0)$ denotes all roots of E_{11} with nonzero power of f_1 . This is a semidirect product of E_{10} and $(f_1)^n \dots (n > 0)$. So, particularly, each unitary representation of E_{10} gives rise to induced unitary irrep of $l_1 \ltimes E_{11}$.

6 Conclusion

In the body of paper we discuss some features of E_{11} approach, which can help in study of different aspects of theory - such as orbifolds, supersymmetry relation, induced representations, etc. We introduce an even subalgebras $Z_2(E_n)$ and find a description of corresponding roots through EE_{11} (for E_{11}), DE_{10} (for E_{10}), and $D_8^{(1)}$ (for E_9). It is proved that for last two cases even roots are completely given by algebras mentioned, for E_{11} that is a hypothesis, supported by computer calculations up to level 142. The possible form of supersymmetry relation (10) is considered. It requires that compact subgroup K_n has a representation the symmetric square of which contains l_1 representation of E_{11} . In view of that we consider the expansion of space of brane charges, i.e. l_1 w.r.t. the $SO(10,10)$ subgroup of E_{11} , and show that first few representations of $SO(10,10)$ in l_1 can be represented in required form. Corresponding relation (28) is a standard 11d supersymmetry relation, decomposed w.r.t. the $SO(10) \times SO(10)$ subgroup. Another result, which may have relation with supersymmetry (and not only) is the finding of Section 3, that second fundamental weight of EE_{11} coincides with the weight of l_1 irrep of E_{11} . Other existing approaches to supersymmetrization of E_{11} are discussed.

Finally, precise answer is obtained for a little group of particle orbit in a semidirect product group $l1 \ltimes E_{11}$, assumed to be an extended symmetry group of E_{11} theory.

In conclusion, we would like to discuss the possible connection of E_{11} approach to well-known method of coadjoint orbits. According to the construction of [1] fields of E_{11}/K_{11} manifold contains both fields and their duals, as is precisely shown for lower level, and corresponding equations of motion are those of generalized self-duality. If one neglects dependence of fields from (infinite number of?) space-time coordinates, that will mean that E_{11}/K_{11} is not a configuration space but rather phase space, since it includes both fields and their conjugate momenta. Taking into account the existence of natural (Kirillov-Kostant) Poisson bracket on the coadjoint orbits of Lie algebras, one can ask whether E_{11}/K_{11} is such an orbit. That would mean that K_{11} is a stabilizer of some element of E_{11} algebra. It is easy to show that it is not the case. Of course, according to the previous discussion it is not a necessary feature, one should take into account a space coordinates, to make a statement precise. We think that application of coadjoint orbit method to E_{11} worths further study.

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References

- [1] P.West, E_{11} and M-theory, hep-th/0104081, Class.Quant.Grav. **18** (2001) 4443-4460
- [2] P.West, Hidden Superconformal Symmetry in M-Theory, JHEP 0002 (2000) 024, hep-th/0001216
- [3] T. Damour, M. Henneaux and H. Nicolai, E10 and a "small tension expansion" of M Theory, Phys.Rev.Lett. 89 (2002) 221601, hep-th/0207267
- [4] E.Cremmer and B.Julia, The SO(8) supergravity, Nucl.Phys. B159 (1979) 141
- [5] B.Julia, "Group disintegration", In S.W.Hawking and M.Rochek, editors, Superspace and Supergravity, Cambridge Univ. Press, 1981
- [6] T. Damour, M. Henneaux and H. Nicolai, Cosmological Billiards, Class.Quant.Grav. 20 (2003) R145-R200, hep-th/0212256
- [7] N.A.Obers and B.Pioline, U-duality and M-theory, Phys.Rept. 318 (1999) 113-225, hep-th/9809039
- [8] H. Nicolai and H. Samtleben, On $K(E_9)$, Q.J.Pure Appl.Math. 1 (2005) 180-204 hep-th/0407055
- [9] J.Brown, S.Ganguli, O.Ganor, and C.Helfgott, E10 orbifolds, hep-th/0409037; J.Brown, O.Ganor, and C.Helfgott, M-theory and E10, Billiard, Branes and Imaginary roots, JHEP 0408 (2004) 063, hep-th/0401053
- [10] H.Mkrtchyan and R.Mkrtchyan, E11, K11 and EE11, hep-th/0407148

- [11] A.Kleinschmidt and P.West, Representations of G_{+++} and the role of space-time, JHEP 0402 (2004) 033, hep-th/0312247
- [12] N.Markus and J.Schwartz, Three-dimensional supergravity theories, Nucl.Phys.B228 (1983) 145-162
- [13] Andre Miemiec and Igor Schnakenburg, Killing Spinor Equations from Nonlinear Realizations, Nucl.Phys. B698 (2004) 517-530, hep-th/0404191
- [14] P. West, E_{11} origin of Brane charges and U-duality multiplets, JHEP 0408 (2004) 052, hep-th/0406150
- [15] P.West, E_{11} , $SL(32)$ and Central Charges, Phys.Lett. B575 (2003) 333-342, hep-th/0307098; A.Kleinschmidt and P.West, Representations of G_{+++} and the role of space-time, JHEP 0402 (2004) 033, hep-th/0312247
- [16] Axel Kleinschmidt and Hermann Nicolai, Gradient Representations and Affine Structures in $AE(n)$, hep-th/0506238
- [17] Axel Kleinschmidt and Hermann Nicolai, E_{10} and $SO(9,9)$ invariant supergravity, JHEP 0407 (2004) 041, hep-th/0407101
- [18] Alex J. Feingold and Hermann Nicolai, Subalgebras of Hyperbolic Kac-Moody Algebras, math.QA/0303179.
- [19] V.G.Kac, Infinite dimensional Lie algebras, Cambridge University Press, 1995.
- [20] B. de Wit and H. Nicolai, Hidden Symmetries, Central Charges and All That, Class. Q. Grav.**18** (2001) 3095-3112, hep-th/0011239.
- [21] Arjan Keurentjes, E_{11} : Sign of the times, Nucl.Phys. B697 (2004) 302-318, hep-th/0402090
- [22] H.Mkrtchyan and R.Mkrtchyan, Little Groups of Preon Branes, Mod.Phys.Lett. A18 (2003) 2665-2672, hep-th/0308065; 10d $N=1$ Massless BPS supermultiplets, Mod.Phys.Lett. A19 (2004) 931-944, hep-th/0312281,